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SR 1 Safety

In our modern society, safety has become much more than just protecting one's well being. Issues around safety have become extremely important to industry, business, governments, and all kinds of institutions including educational institutes. The understanding and application of safety in a broad sense has become an industry in itself. Today, even to be considered for many jobs, one must take safety courses or have a variety of safety-training certificates. Now is the time for you to continue developing an attitude and awareness of safety.

Safety is everyone's responsibility. The Provincial Government, the local school board, your teachers, and you all have an important role in keeping a safe environment. Alberta Education has prepared a detailed document, "Safety in the Science Classroom", outlining safety roles and responsibilities, and providing extensive information on potential hazards and safety procedures. This document is available online; go to www.pearsoned.ca/school/physicssource and follow the link to *Safety in the Science Classroom*. Of particular interest to you in this physics course is Chapter 6: Physical Hazards. For more technical information on issues, follow the link to *Health Canada Index* and search the topic alphabetically.

In general, your role in maintaining safety is to act responsibly by carefully following directions, learning how to recognize potential safety hazards, and how to respond to potentially unsafe situations and emergencies. If you are unsure about how to proceed, ask your teacher.

The Canadian Hazardous Products Act requires chemical manufacturers to include all hazard symbols and the degree of hazard on product labels. You may recognize hazard symbols on many household products. These symbols may indicate hazard(s), precaution, and first-aid treatment.

Hazardous Product and WHMIS Symbols

Household hazardous product symbols indicate the type of danger and the degree of danger. They appear in either a triangle (which means "caution"), a diamond (which means "warning"), or an octagon (which means "danger"). There are also numerous laboratory and industry hazard symbols in use.

Some symbols relevant to Physics 20 and 30 are shown below:

Flammable Hazard: The material could ignite (catch on fire) if exposed to flames, sparks, or friction.
Explosive Hazard: The material or equipment could explode.
Toxic Hazard: The material is very poisonous and could have immediate and serious effects.
Corrosive Hazard: The material may corrode ("eat away at") clothing, skin, or other materials.
Biological Hazard: Be alert to the possibility of poisoning or infection from microscopic and other organisms.
Electrical Hazard: Be alert to the possibility of an electric spark or shock.
Hot Surface: Skin or clothing may burn if touched.
Laser: Dangerous to eyes.
Radioactive Material: May pose hazard for short or long term exposure.

▲ **Figure SR 1.1** Laboratory and industry hazard symbols

Many of the chemical products used in Canadian schools are manufactured in the United States. To standardize the labelling systems, WHMIS (the Workplace Hazardous Materials Information System) was developed. The symbols belonging to this system appear on materials and products used both in workplaces and our schools.

▲ Figure SR 1.2

Laboratory Safety

Approach all investigations, especially in the laboratory, with maturity. Before you begin, read the instructions carefully, noting all safety precautions. In addition, your teacher may provide other safety reminders and rules pertaining to the laboratory activity. It is your responsibility to inform your teacher of medical conditions such as possible allergies to materials used (e.g., latex) or by-products of the activity. Inform your teacher if you wear contact lenses.

1. General Precautions and Safety Equipment

a) Identify all safety equipment in the laboratory.

b) Know the location of and how to operate safety equipment, including the fire extinguisher, fire blankets, eyewash fountains, sand, and the first-aid kit.

c) Know how and where to get help if needed.

d) Wear appropriate laboratory apparel, which may include safety goggles, gloves, and/or lab aprons.

e) Tie back long hair and secure any loose clothing.

2. Precautions with Mechanical Hazards

a) Rotating machinery or moving devices can catch loose clothing, fingers or hair; therefore, keep a safe distance away from moving parts.

b) Strong magnets can snap on ferromagnetic materials and other magnets very quickly. Use caution to avoid pinching skin or cutting clothing.

c) Projectile launchers should be used only with appropriate eye protection and a clear "line-of-fire." Be aware of the potential for a misfire or backfire.

d) Model rockets with air, water, or solid-fuel motors can be a hazard. Wear eye protection and stay well clear of the launch area and potential trajectory. Make sure everyone watches for rocket parts falling back to the ground.

3. Precautions with Electrical Sources

a) Do not use 110-V AC equipment if it has a damaged plug (e.g., missing the ground pin) or a frayed cord. Always disconnect the cord from the socket by pulling the plug, not the cord.

b) Keep water and wet hands away from electrical cords.

c) Do not touch a person in contact with live electrical currents. Disconnect the power source first. Then give artificial respiration if necessary. Call for help and treat burns.

d) Make sure electrical cords are not placed where someone could trip over them.

e) Do not allow a short circuit connection to a dry cell or battery. Dangerous amounts of heat can be generated in the wires and in the cells themselves, potentially causing an explosion or fire.

f) Never attempt to recharge a non-rechargeable battery. Never cut open batteries. Their contents can be corrosive and poisonous.

g) Keep flammable liquids away from electrical equipment. Sparks, in a motor for example, could ignite flammable vapours.

h) Spark timers create a very short but high voltage spark, which can give a minor electrical shock to anyone who touches a "live" part of the circuit. Although the shock itself is not dangerous, the surprise and sudden reaction can cause elbows to fly or objects to be dropped.

i) Some high voltage devices can cause nasty shocks or skin burns. Be aware of the potential danger of charged capacitors, tesla coils, electrostatic generators, and transformers. Use only under the guidance of your teacher.

j) When hooking up circuits, always have your teacher check the circuit before turning on the power.

4. Precautions with Electromagnetic Radiation

a) Never look directly into an infrared (IR) or an intense, visible ultraviolet (UV) light source. Intense light can harm the retina. UV and IR radiation are absorbed by the cornea and eye contents, and can cause burning and overheating or other damage.

b) Never look directly into the beam of an operating laser, even one with a low power. The eye focuses the laser light onto the retina, resulting in a power density of about 50 times that of direct sunlight. This can cause pinpoint burns to the retina.

c) Guard against stray reflections and turn the laser off when not in use.

d) Use radioactive sources only under the direction of your teacher.

e) In all cases, the potential for harm from radiation increases with exposure. Exposure can be minimized by limiting the time of use and maximizing the distance away from the source.

SR 2 The Inquiry Process

Inquire: to seek knowledge of, to ask about, to investigate, or to seek information by asking.

Have you ever seen "sun dogs"? They are a common occurrence in Alberta, especially in the winter. Often they are coloured and associated with a coloured ring, or halo, around the sun. They seem to occur in thin clouds or in frosty air. You may have noticed that they are always the same distance from the sun and the colours are always in the same order: red closest to the sun and violet farther away. This set of observations can be the focus of an inquiry process.

▲ **Figure SR 2.1** Prairie sun dogs

The inquiry process is a model of learning incorporated in Alberta Education curricula; it is not a separate topic or option. The inquiry process is applicable to all learning, and is especially suited to learning physics. Learning an inquiry approach is more than a way to succeed in physics; it is a useful way to deal with problems and challenging situations throughout any future career.

The inquiry process is non-linear (there may be some side-tracks or dead ends), flexible (you can bend the rules or the process), individual (you can develop your own process), and recursive (you will need to revisit or loop through parts of the process as you go). An inquiry process model contains six components that connect together, all around your own thinking or reflection on the process. These components are:

- Planning
- Retrieving
- Processing
- Creating
- Sharing
- Evaluating

Planning

In order to inquire, you must have something in mind about which to inquire. Normally, the planning stage involves recognizing a situation, event, topic, or occurrence for which there is some unknown component. This leads to questions. In the planning stage, you (or your teacher) will need to look at the situation at hand and ask a question to be investigated. You may have many questions, but part of the process is to reflect on the situation and narrow (or in some cases, broaden) your question so that it is something you can actually investigate. From there, you will need to develop your process to lead through the other stages of the inquiry process. One way of working through the planning stage is to ask yourself the following questions:

- What do I want to know?
- What do I think the result might be?
- How can I find out?
- What do I need to do to find out?
- How will I know when I have found out?
- What form will my final results take?
- How can I best share my results with others?
- How can I evaluate what I have done?

In this stage, you will need to develop a clear inquiry question, propose a thesis or hypothesis, identify variables or related factors, create a data or information gathering process (experiment or research strategy), and recognize where your results may end up. Sometimes, this is the most difficult or lengthy part of the entire process, but very important.

Retrieving

Once you have an inquiry question, a hypothesis to test, and a plan to follow, you can begin the process of retrieving. This may involve experimentation or research. You may be gathering text information or numerical data from measurements. You may be using several different data sources including your own experimental data. In this stage, you may need to revisit *Planning* if you find difficulties or discrepancies during your information gathering.

Processing

The information or data you retrieve must be processed. You must evaluate the information, sort good data from poor data, decide what data is relevant, and perhaps re-identify variables. At this stage, you may find yourself looping through parts of the *Planning* and *Retrieving* stages again. Once you have good information, you need to decide how to display and analyze the data. What combination of data tables, graphs or graphic organizers should you use? Is there some mathematical analysis such as finding a slope, modelling a curve with an equation, or some statistical calculations that will be useful? Again, you may want to loop back to *Planning*, or ahead to *Creating* and *Sharing* to decide what detail of analysis you want to perform on your information. Ultimately, you need to be able to answer the question, "What does this information mean?"

Creating

You have asked a question, retrieved your information, and processed it. Now you must put the package together by creating a final product. Remember to look ahead to *Sharing*. In this product, you must clearly indicate your initial inquiry, provide a summary of your data or information, explain the meaning of your data, and state some conclusions regarding your inquiry question. At this point, you may find that you have more questions. This could lead you to another inquiry, and another, and another. This is the essence of how scientific knowledge continues to grow.

Sharing

This stage in the inquiry process is often devalued, yet it is a crucial step in the process. In science, if a new discovery is not communicated, then it is lost. In education, communicating ideas to others is one of the best learning processes. You don't really know or understand something until you can share the ideas with others. How you share will depend in part on how you created your final product. There are many modes of sharing: oral presentation, poster display, written report, demonstration, art work, working model, skit… Your job will be to choose a method that best fits you, your results, your classroom situation, and your audience. Again, you may need to loop back to *Planning* and *Creating* to get this in the most suitable form.

Evaluating

Now is the time to look at the whole process (not just the results or answers). Ask yourself these questions:

- What worked well?
- What became a challenge?
- Is there another or better way of doing any one of the stages in this process?
- What parts of the process were easier or more difficult, or more or less effective?
- How would I coach someone else to do this same inquiry in a more efficient way?
- Are there other questions or situations that might be resolved by the process I followed?
- What have I learned (about the inquiry question and about learning)?

By critically evaluating what you have done, you will learn process skills beyond physics or science; you will develop skills to last a lifetime.

A skeleton of a sample inquiry:

Planning

Situation: a solar halo display is very clear and colourful in the sky.

Question: Where do the sun dog's colours come from?

Hypothesis: This is the same effect as the rainbow. If I spray water into sunlight, then I should be able to see a halo.

What to do: Research rainbows and halos in print and online. Do an experiment with light rays and water drops to try to create a halo.

How do I know I have the answer: I can create a halo and a rainbow and match them together.

My final results: I will have a poster display with photos I have taken, and I will explain the results to my classmates.

Retrieving

Internet search informs me that halos and rainbows are different. From photos, I see that the colours are reversed. There are different types of halos. Sun dogs are part of ice crystal halos caused by refraction.

Revisit planning

Question: Does refraction of light through ice or other transparent solid crystals model the position of colours in sun dogs?

Hypothesis: If I shine a white light beam on a transparent solid crystal, then I will be able to see colours located in different directions.

Experiment: Shine light through ice crystals, prisms, or other transparent solids, and look for colours in different directions.

Processing

Can I see colours produced by light passing through the crystals? Is there a way for me to quantify my observations, e.g., can I measure directions?

To what degree of accuracy can I measure directions? Do I have enough data? Do I need to look at more variables such as the shape or material of the crystal? How can I record my observations in the most meaningful way for this context?

Creating

I will summarize my specific question and investigative process. My results include: data tables showing the approximate angles where I see different colours of light refracted through prisms of different shapes; three photos of my apparatus set-up and colours I could see; and internet photos of more sophisticated experiments. I will also create diagrams of what could be seen in my apparatus according to theory, and a poster display of the theory behind the experiment and actual halos. There are other questions arising from my work that I will pose on my poster. For example: What happens to the halo when an ice crystal tips on its side?

Sharing

I will give a short presentation to my class to explain my question and results, and model the path of light with white and coloured yarn through a foam block. Classmates will be able to see "colours" only at certain directions.

Evaluating

I will look critically at the entire process once I am finished.

SR 3 The Problem-Solving Process: GRASP

Solving Numerical Problems

A significant amount of effort in physics is spent in solving problems that have a numerical component. Often these problems seem more difficult than they really are because they involve physics concepts, principles, and/or laws as well as mathematical operations. Research into the problem-solving abilities of professionals and novices shows that professionals have logical procedures they follow when solving problems while novices who are having difficulty do not. The more methods the problem-solver can apply, the more adept he/she is at problem solving.

The approach used in numerical problems throughout the textbook follows four basic steps. These steps are easy to remember and apply because combined, the first letters of the key words spell GRASP. A description of the four steps is provided below.

Step 1: List what is Given

The first step in solving numerical problems involves answering the question, "What information are we given?" This is sometimes referred to as *data extraction*. To answer this question, read the problem carefully, study the information given, and represent physical quantities and numerical data with appropriate symbols, units, and directions (if necessary). Write the data in scientific notation to the correct number of significant digits (SR 6.3).

Step 2: List what you are Required to find

The second step involves answering the question, "What am I required to find?" To answer this question, identify what the problem is asking you to do. Be sure to note the units requested, if specified, and, for vector quantities, the direction. Answering this question will point you in the right direction and prevent you from being distracted by irrelevant information.

Step 3: Analysis and Solution — Analyze the problem carefully and work out the Solution

The third step requires a careful analysis of the problem before solving. To analyze the problem you must break it down into a series of logical steps. Begin by sketching a diagram. Many physics problems lend themselves to a diagram and the diagram often provides the key to solving the problem. Write down all the relationships you know involving the givens and the required. Also, write down any assumptions that must be made in order to solve the problem. An assumption is anything that must be taken for granted. Next, start with what you are trying to find, and answer the question, "What additional information do I need to calculate the unknown?" This may be a constant that you have to look up in a reference book or from a table of constants given in the textbook. Organize and sequence the information you have to form the solution. In physics, this often involves substituting appropriate data into an equation. It is good practice to rearrange an equation to solve for an unknown variable in terms of the other variables before substituting to obtain the final answer. Always be on the lookout for errors in the mathematical computations and check that the answer has the correct number of significant digits, and that appropriate units are included.

Step 4: Paraphrase the solution

The numerical answer should be stated in a form that answers the original question. Since the original question was a sentence, the statement of the final answer should also be a complete sentence. Physical quantities should include units and directions, if appropriate.

You can use the following Numerical Problem Checklist to guide your work.

Numerical Problem Checklist

Given

- ☐ read problem carefully
- ☐ extract data
- ☐ represent physical quantities with appropriate symbols
- ☐ include units with physical quantities
- ☐ include directions where needed
- ☐ show the correct number of significant digits

Required

- ☐ identify what the problem is asking for
- ☐ identify units of the final answer
- ☐ identify direction of the final answer (for vector quantities)

Analysis and Solution

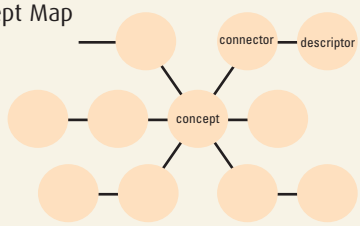
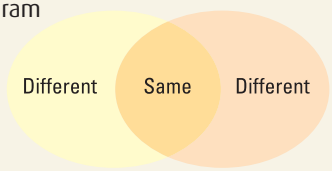
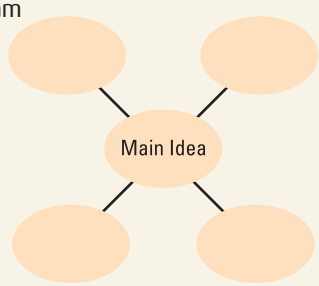
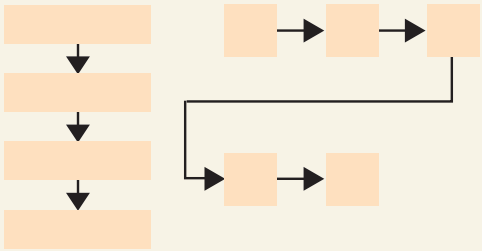
- ☐ draw a sketch
- ☐ write down possible relationships
- ☐ list viable assumptions
- ☐ write an appropriate equation containing the needed unknown variable and only other variables that are known or can be found from the given data
- ☐ identify and look up constants needed
- ☐ identify inconsistent units and perform needed unit conversions
- ☐ identify "red herrings" (i.e., extra bits of numerical information not needed to solve the problem)
- ☐ note the least number of significant digits in the given data
- ☐ note directions (for vector quantities)
- ☐ rearrange the equation to solve for the unknown
- ☐ substitute data into the rearranged equation
- ☐ simplify the mathematics: solve for the numerical answer and simplify the units
- ☐ check the mathematical calculations
- ☐ check the number of significant digits
- ☐ check the direction (if required)

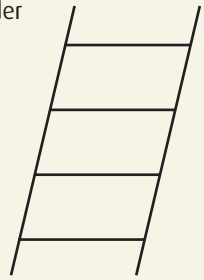
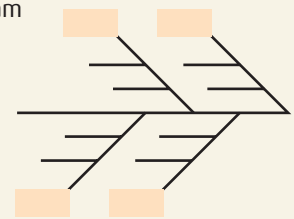
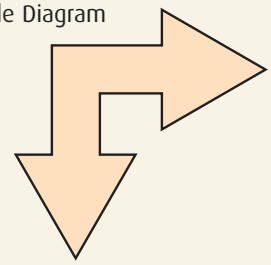
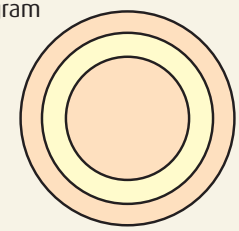
Paraphrase

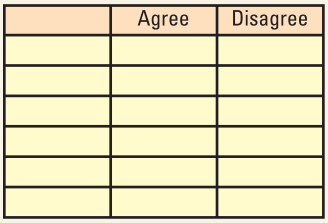
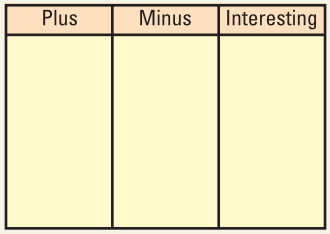
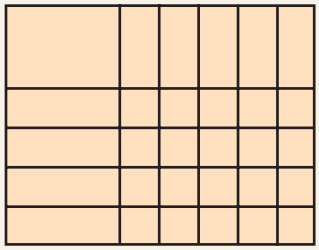
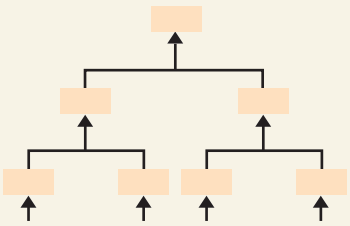
- ☐ write the final answer in a complete sentence
- ☐ check that units are included with numerals
- ☐ check accuracy of significant digits
- ☐ check accuracy of direction (if required)
- ☐ check that the original question has been answered and that the answer seems realistic

SR 4 Using Graphic Organizers

Graphic organizers are effective tools that can help you learn. They enable you to solve problems and think critically, through analyzing similarities and differences, inferring sequences, and establishing cause-and-effect relationships. They generate discussion and negotiation of ideas, extend comprehension of a concept, theme, or topic, and lead to organized representation and presentation of understandings. You can use them to brainstorm, demonstrate what you know, and organize your thoughts before planning a presentation or writing a report or essay. The following chart outlines a number of graphic organizers, their intended purposes, and how to use them as you study science.

Type of Graphic Organizer	Purpose	Method
Concept Map 	Used to clarify relationships and linkages between concepts, events, or ideas	Brainstorm ideas and link together from "big to small" with arrows or lines linking words.
Venn Diagram 	Used to visualize similarities and differences between two or more ideas, topics, or concepts	Brainstorm similarities, and list these in the overlapping section of the two circles. Then brainstorm differences and list these in the non-overlapping sections.
Web Diagram 	Used to clarify concepts and ideas by clustering them	Cluster words and/or information around a central object, concept, or idea.
Pie Chart	Used to estimate the relationship of parts to the whole	Estimate/research the importance or amount of proportionate time of each aspect of an event in relation to the whole.
Flowchart/Sequence Chart 	Used to map out your thinking about an issue or to organize ideas for an essay or report	Brainstorm aspects of the whole event. Select important aspects and put them into sequential order.

Type of Graphic Organizer	Purpose	Method
Ranking Ladder 	Used to rank ideas in order of importance	Brainstorm ideas and rank them in order from least important (bottom rung) to most important (top rung).
Fishbone Diagram 	Used to identify cause-and-effect relationships	Identify a problem to be solved. List the "effect" at the head of the fish. Brainstorm "possible causes" in each bone. Rank the causes and circle the most probable ones, justifying your choice.
Right-Angle Diagram 	Used to explore the consequences of an idea and the impact of its application	Briefly describe the idea you are exploring on the horizontal arrow. Brainstorm consequences of the idea, and list these to the right of the horizontal arrow. Expand on one consequence, and list details about it along the vertical arrow. Describe social impacts of that trait below the vertical arrow.
Target Diagram 	Used to weigh the importance of facts and ideas	Brainstorm facts and ideas. Rank their importance and place the most important facts or ideas centrally, and the least important ones toward the outer rings.

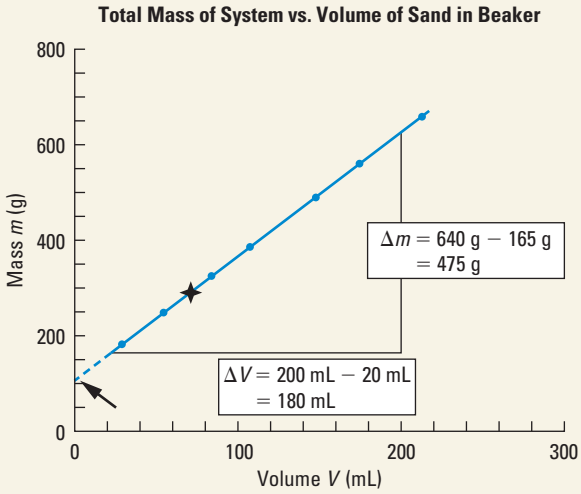
Type of Graphic Organizer	Purpose	Method
Agree/Disagree Chart 	Used to organize data to support a position for or against an idea or decision	List a series of statements relating to a topic or issue. Survey agree-disagreement before discussion. Survey again after discussion and research.
PMI (Plus, Minus, Interesting) Chart 	Used to summarize the positive and negative aspects of a topic or issue, as well as identify interesting aspects of the topic for possible further research	Sort ideas or information about a topic or issue in a three-column chart that has the following headings: Plus (+), Minus (−), and Interesting.
Gathering Grid 	Used to make distinctions between ideas or events	Gather information on a number of ideas or events and arrange it on a grid. Each idea or event is assigned to a separate row. Analyze the information according to selected criteria in each specific column.
Concept Hierarchy Diagram 	Used to identify and sequence the subordinate concepts needed to understand a higher-order concept	Place the higher-order concept at the top of a page. Then consider the question, "What concepts need to be understood before the higher-order concept above can be grasped?" The same question is then asked for each of the subordinate concepts identified and a hierarchy of connected concepts is created.

SR 5 Graphing Data

5.1 Graphing Techniques

▼ **Table SR 5.1** Total mass of system as sand is added to a beaker

Total Volume of Sand Added *V* (mL)	Total Mass of System *m* (g)
28	188
55	258
84	333
106	391
148	500
174	567
210	661



▲ **Figure SR 5.2**

Physicists make extensive use of graphs to convey information and to help determine how one physical quantity is affected by another. To review simple graphical analysis techniques, as an example, use the data from a simple measurement experiment where students added given volumes of sand as measured in a graduated cylinder to a beaker on a balance, recording the total volume of sand in the beaker and the total mass of the system as shown on the balance.

The Data Table

A data table is the most practical way to record quantitative data. Table SR 5.1 above shows the data from the student experiment of adding sand to a beaker on a balance. Note that the name of each variable, the symbol, and the unit of measurement (in round brackets) are recorded at the top of each column.

The Title of the Graph

Figure SR 5.2 shows a sample graph for a student's experiment. Every graph needs a title to describe what it is about. The title is placed at the top of the graph or in a box on a clear area above the graph.

The Axes of the Graph

Plot the independent variable on the horizontal *x*-axis and the dependent variable on the vertical *y*-axis. The variable that is changed intentionally is called the *manipulated* or *independent variable*. Volume of sand in the beaker was the manipulated or independent variable in the experiment as students chose how much to add for each trial. The mass of the system depended on how much sand was added, thus the mass was the responding or *dependent* variable.

Label each axis with the name, symbol, and unit of the variable being plotted, as shown in Figure SR 5.2. Scales are chosen for each axis to spread the measured values across the graph paper without making the plotting difficult. The maximum values in the data table determine the maximum numbers on the scales of the axes.

To set a scale for an axis, analyze the data to be plotted and choose increments appropriate to the data.

In Table SR 5.1, the maximum volume value was 210 mL, and the minimum volume value was 28 mL. Increments of 20 mL on the *x*-axis would be appropriate for the data. As well, the maximum total mass of system was 661 g, and the minimum total mass of system was 188 g. Increments of 50 g on the *y*-axis would be appropriate for the data.

Plotting the Data and Drawing the Line of Best Fit

Use a pencil to plot the data points as accurately as possible by making a small visible dot. Accuracy is important. Use the actual data values and make your best estimate of values within the scale grid. Once all of the data points have been plotted, a line of best fit is drawn. A line of best fit is a line that shows the trend of the points. Make the smoothest curve you can, balancing points that do not fit the curve evenly above and

below the curve. Do not try to have the curve or straight line go through all the dots since most data points have some error. The scatter of the data points from the smooth line indicates the extent of the errors in the data.

Where a point is far off the line, a serious error may have been made. If this occurs, measure the data for that point again, if possible. If you believe an error was made, still plot the point, but ignore it while drawing your best fit curve.

Interpolating from the Graph

Interpolation is the process of estimating a value that is between two directly measured data points of a variable. To interpolate, first locate the point on the appropriate axis for the value of the variable in which you are interested. Next, draw a line perpendicular to this axis to intercept the line of best fit. From this point on the line of best fit, draw a second line perpendicular to the second axis. Read the value of the second variable from this axis. For example, in Figure SR 5.2, a volume of 70 mL of sand is interpolated to a total mass of 300 g (indicated by the small star).

There is some risk of inaccuracy involved in interpolation, since it is assumed that the trend of the line continues between the measured points. This assumption is not always valid.

Extrapolating from the Graph

Extrapolation is the process of estimating the values of a data point beyond the limits of the known or measured values. However, there is a considerable risk of inaccuracy, because it is assumed that the trend of the curve continues outside the range of the data. When the line is extended, a dotted line is used to show that the extension is little more than guesswork. The arrow in Figure SR 5.2 shows the process of extrapolating the curve to find the mass of the system without any sand in the beaker. What is this value and what does it represent? How valid is the value?

Calculating the Slope

If the line of best fit is straight, the *slope* of the line can be found. The slope of the line is defined as the rate of change of one variable with respect to the other and is found by the ratio of the rise to the run. To find the slope, find two points far apart on the line of best fit whose values are easily readable from the scales on the axes. On the graph, lightly draw a horizontal line from the lower point and a vertical line from the higher point so that they intersect. Use the axis scale to determine the change in vertical value (rise) and change in horizontal value (run) along these two line segments.

In Figure SR 5.2, the rise is shown as 475 g and the run as 180 mL.

The slope of this graph is thus:

$$\text{slope} = \text{rise/run} = 475\ \text{g}/180\ \text{mL} = 2.64\ \text{g/mL}$$

Notice that the slope in this example has units; it also has some physical significance: it represents the density of the sand.

Writing the Equation of the Line

If the trend of the curve is a straight line, then changes in the plotted variables are directly proportional to each other. As the change in one variable doubles, the change in the other doubles, and vice versa. The general equation for a straight line is $y = mx + b$, where y is the variable on the vertical axis, x is the variable on the horizontal axis, m is the slope of the line, and b is the vertical axis intercept.

Figure SR 5.2 shows a straight line for volumes of sand at least up to 210 mL. For this range, the change in mass is directly proportional to the volume of sand added. The general equation for the linear graph in Figure SR 5.2 is: $m = \rho V + \text{b}$ where m is the total mass of the system, ρ is the density of the sand, V is the total volume of sand in the beaker, and b is the mass of the empty beaker. The specific equation for this graph is:

$$m = (2.63\ \text{g/mL})V + 115\ \text{g}.$$

Using the Equation of the Line

It is often more convenient to extrapolate or interpolate from the specific equation than from the graph. For example, the total mass of the system could be determined if 800 mL of sand is added, even though our beaker may not be that big and our graph does not extend that far. To do this, substitute 800 mL for V in the equation and calculate m. The accuracy of this result depends on the accuracy of the equation, which in turn depends on the accuracy of the determination of slope and vertical axis intercept.

5.2 Using the Graphing Tools

Graphing calculators make the process of plotting and interpreting graphs easy and efficient. Data from an experiment can be entered into the calculator and displayed as a scatterplot. The calculator can be used to determine the function that best models a given scatterplot. The information provided for this function can also be used to write the equation that best describes the relationship between the two plotted variables.

The graphing calculator can also be used to help us explore the graph of a given equation or relationship. It can be used to interpolate values between the plotted points or to extrapolate values beyond the plotted points. These uses make the graphing calculator a very powerful laboratory tool.

Data can also be stored and plotted in a computer spreadsheet such as Microsoft Excel.

The eMath activities in this textbook provide opportunities for you to use the graphing calculator or a computer spreadsheet.

SR 6 Math Skills

6.1 Measurement: Accuracy and Precision

Measurement is a process of comparing some unknown characteristic, attribute, or quantity to some known or accepted scale or standard. Measurement involves tools and technique. If there is a problem with either, there can be problems with the quality of the measured data. A tape measure with the end broken off would give incorrect measurements if the problem was not noticed. Using a metre stick to measure the thickness of a hair would not work well. Forgetting to include the weight of the fuel when weighing in freight to load a plane could be disastrous. At best, a measurement is an estimate; there is always some amount of uncertainty to the value you record. To make a measurement, and to use measured values correctly, we must understand a number of important issues.

Accuracy

Accuracy is a means of describing the quality of measurements, or how closely a measurement agrees with the accepted or actual value of the quantity being measured. A broken tape measure will not give accurate values for length. The difference between an observed value (or the average of observed values) and the accepted value is called the *deviation*. The size of the deviation is an indication of the accuracy. Thus, the smaller the error, the greater is the accuracy.

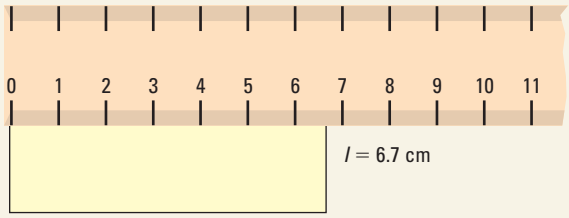
The *percent deviation* is determined by subtracting the accepted value from the measured value, dividing this by the accepted value, and multiplying by 100. Thus,

$$\text{percent deviation} = \left| \frac{(\text{measured value} - \text{accepted value})}{\text{accepted value}} \right| \times 100\%$$

Precision

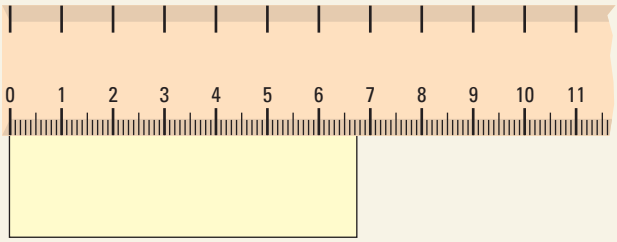
Precision is the degree of repeatability of measurements; it depends on your care and technique. If the same measurement is carefully made several times independently, we find that we may get variations in the last digit we read. This limitation defines the precision of the measurement. The precision of a measuring instrument depends on how finely the scale is divided. A ruler with mm divisions will not be useful in measuring the difference in hair thickness between blondes and red-heads. The smaller the divisions of its scale, the less uncertainty there will be in reading values.

Any measurement that falls between the smallest divisions on the measuring instrument is an estimate. We should always try to read any instrument by estimating tenths of the smallest division. For a ruler calibrated in centimetres, this means estimating to the nearest tenth of a centimetre, or to 1 mm. Using this procedure, the length of the object in Figure SR 6.1 is found to be 6.7 cm. We are certain of the 6, but the 0.7 is an estimate. In reality, it could easily be 0.6 or 0.8. It is however, unlikely that it would be 0.5.



▲ **Figure SR 6.1**

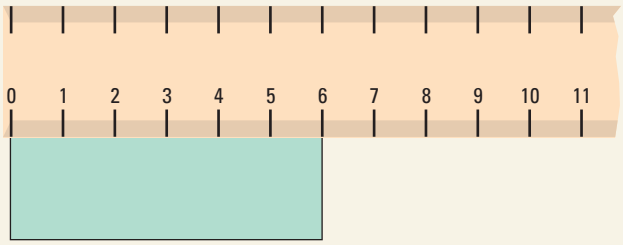
Figure SR 6.2 shows the measurement of the same object using a ruler calibrated in millimetres. The reading estimated to the nearest tenth of a millimetre appears closest to 6.74 cm. It might be tempting to record the length as either 6.7 cm or 6.8 cm. This would be wrong. We can tell that the length is between the two divisions. The estimated digit is always shown when recording the measurement. The estimated digit in this reading is 0.04 cm.



▲ **Figure SR 6.2**

Figure SR 6.3 shows a different object being measured with a ruler calibrated in centimetres. The length falls exactly on the 6-cm mark. Should the length be recorded as 6 cm or 6.0 cm? Remember that with a centimetre ruler we can estimate to tenths of a centimetre. With this ruler we can therefore distinguish readings of 5.9 cm

and 6.1 cm. The object is right on a division mark, so the estimated digit is zero-tenths of a centimetre. Zero-tenths is indicated by 0.0 and the correct reading is 6.0 cm, not 6 cm.



▲ **Figure SR 6.3**

Indicating the Precision of Measured Quantities

The *precision* of a measurement is indicated by the number of decimal places. For example, 2.861 cm is more precise than 581.86 cm even though the latter contains more digits. This is because the three decimal places in 2.861 make it precise to the nearest one-thousandth of a centimetre, while the two decimal places in 581.86 make it precise to the nearest one-hundredth of a centimetre. In this physics program, angle measures will have a precision no better than 0.1°.

Significant Digits

The accuracy of a measurement is indicated by the number of significant digits. *Significant digits* are the specific number of digits used to communicate the degree of uncertainty in a measurement. When we are expressing a physical quantity as a number, how many significant digits should we indicate? These rules should help you decide.

- Numbers obtained by counting are considered to be exact and contain an infinite number of significant digits. For example, if there are 12 stopwatches in a classroom, there are not 11 stopwatches, or 13 stopwatches, or 12.35 stopwatches. There are exactly 12.000… stopwatches. The zeros may be extended to as many decimal places as necessary in calculations.
- Numbers obtained from definitions are considered to be exact and contain an infinite number of significant digits. For example, 1 m = 100 cm, and 1 kW·h = 3600 kJ, are definitions of equalities. π ($\approx$ 3.141 592 654) has an infinite number of decimal places, as do numbers in formulae such as $P = 4s$, where P is perimeter of a square, and s is the length of a side.
- All of the digits from one to nine (1, 2, 3, … 9) are significant, so 424.7 m or 0.4247 km each have four significant digits.
- All zeros to the left of the first non-zero digit are not significant. For example, 1.4 kg and 0.0014 kg each have two significant digits.
- Zeros between other non-zero digits are significant. Therefore, 501.009 *s* has six significant digits.
- Any zero to the right of a non-zero digit is significant. Therefore, the mass of an object written as 2000 kg has four significant digits. If the mass is a stated value, not a measured value, we can indicate that we know it to four significant digits by using scientific notation and writing the mass as 2.000×10^3 kg.

Good measurements have high degrees of both accuracy and precision. In order to assure better accuracy, instruments should be calibrated. *Calibration* involves making sure the scale divisions are spaced properly and that the zero reading is correct. Better precision is attained by using better tools, those with finer scales. Good technique, of course, is also necessary for both precision and accuracy.

6.2 Mathematical Operations with Data

When doing calculations with measured values, never keep more digits in the final answer than in the least accurate number in the calculation. For example, 0.6 + 0.32 = 0.9, not 0.92. This procedure for using only meaningful digits is called rounding off. The procedure for *rounding off* digits is as follows:

- When the first digit discarded is less than five, the last digit retained is left the same. Notice that we start rounding off at the digit immediately after the last digit we are retaining. For example, 14.248 kg rounded to three digits is 14.2 kg, since the fourth digit (4) is less than five.
- When the first digit discarded is a five or greater, we increase the last digit retained by one. Therefore, 7.8361 km rounded to three digits is 7.84 km, and 4.255 01 s rounded to three digits is 4.26 s.
- Consider numbers that are exact counts to be perfectly precise. For example, the average mass of three cars having masses of 1000 kg, 1250 kg, and 1165 kg is (1000 kg + 1250 kg + 1165 kg)/3 or 1138 kg. The denominator (3) in this example is an exact count, and therefore the answer includes four significant digits.

- Consider fractions and defined equalities to be perfectly precise. The fraction $\frac{1}{2}$ in the equation $E_k = \frac{1}{2}mv^2$ does not influence rounding off. Neither does the defined equality 10 mm = 1 cm.

Note: In the examples in this textbook, intermediate steps are shown rounded off to one extra significant digit. In reality, all digits are carried through the calculations until the final answer is reached, at which point the final answer is rounded off appropriately.

Rules for Significant Digits in Mathematical Operations

Adding or subtracting: the precision, as shown by the number of decimal places in the values being used, determines the number of significant digits in the answer. Round off the answer to the same precision as the least precise value used. For example, 11.2 kg + 0.24 kg + 0.336 kg = 11.776 kg, is rounded off to 11.8 kg because the least precise value, 11.2 kg, is only given to the first decimal place.

Multiplying or dividing: the value with the least number of significant digits determines the number of significant digits in the answer. For example, a distance of 34.28 m is travelled in a time of 4.8 s. The average speed is calculated by (34.28 m)/(4.8 s) = 7.141 666 m/s, rounded off to 7.1 m/s to two significant digits because the time of 4.8 s has only two significant digits.

Note: All quantities must have compatible units or prefixes before being used in calculations.

6.3 Exponential Notation and Scientific Notation

Exponential Notation

Exponential notation makes use of powers of ten to write large and small quantities and to convey the number of significant digits. The first part of the number is called the *coefficient,* and the power of ten is the *exponent*. The radius of Earth may be written in exponential notation to three significant digits as 638×10^4 m, 63.8×10^5 m, 6.38×10^6 m, or 0.638×10^7 m. The diameter of a typical atom may be expressed to one significant digit as 1×10^{-8} cm or 0.1×10^{-7} cm.

Scientific Notation

Any measurement that consists of a coefficient multiplied by a power of ten is expressed in exponential notation. Both 6.38×10^6 and 0.638×10^7 are in exponential notation. Scientific notation is a special kind of exponential notation. For a number to be in scientific notation, the coefficient must be greater than or equal to 1, and less than 10. This means that 6.38×10^6 is expressed in scientific notation and 0.638×10^7 is not.

Scientific notation enables us to show the correct number of significant digits. Remember that any zero to the right of the decimal point is significant. Therefore, if all four digits in the measurement 3400 J are significant, then it would be written in scientific notation as 3.400×10^3 J. However, if only two digits are significant, it would be as 3.4×10^3 J.

The results of all calculations should always be expressed in scientific notation, unless you are told otherwise. This involves moving the decimal point and changing the exponent until the coefficient is between 1 and 9. The exponent is decreased by one for each position the decimal point in the coefficient is moved to the right, and increased by one for each position the decimal point is moved to the left. For example,

a) $500 \times 10^4 = 5.00 \times 10^2 \times 10^4$
$= 5.00 \times 10^{2+4}$
$= 5.00 \times 10^6$

b) $0.068 \times 10^{-3} = 6.8 \times 10^{-2} \times 10^{-3}$
$= 6.8 \times 10^{-2+(-3)}$
$= 6.8 \times 10^{-5}$

Exponential Notation and Mathematical Operations

Multiplication

The product of exponential numbers is determined by multiplying the coefficients and adding the exponents. For example,

$(3.0 \times 10^2)(4.0 \times 10^{-6}) = (3.0 \times 4.0)(10^2 \times 10^{-6})$
$= 12 \times 10^{2+(-6)}$
$= 12 \times 10^{-4}$
$= 1.2 \times 10^{-3}$

Division

To divide numbers written in scientific notation, the coefficients are first divided, and then the exponent in the denominator is subtracted from the exponent in the numerator. For example,

$$\frac{3.3 \times 10^5}{6.6 \times 10^{-2}} = \frac{3.3}{6.6} \times \frac{10^5}{10^{-2}}$$
$$= 0.50 \times 10^{5-(-2)}$$
$$= 0.50 \times 10^{5+2}$$
$$= 0.50 \times 10^7$$
$$= 5.0 \times 10^6$$

Addition and Subtraction When the Exponents are the Same

When the exponents are the same, the coefficients are added or subtracted as in normal arithmetic. The exponent in the final answer remains the same. For example,

$$(2 \times 10^{-4}) + (3 \times 10^{-4}) - (1 \times 10^{-4})$$
$$= (2 + 3 - 1) \times 10^{-4}$$
$$= 4 \times 10^{-4}$$

Addition and Subtraction When the Exponents are Different

When the exponents are different, the numbers must first be converted to a form in which all exponents are the same. The decimal point is moved so that all have the same exponent as the largest number in the group. Then, the coefficients are added or subtracted accordingly. The exponent in the answer remains the same as the largest exponent. For example,

$$(1.00 \times 10^{-3}) - (2.00 \times 10^{-4}) + (400 \times 10^{-5})$$
$$= (1.00 \times 10^{-3}) - (0.200 \times 10^{-3}) + (4.00 \times 10^{-3})$$
$$= (1.00 - 0.200 + 4.00) \times 10^{-3}$$
$$= 4.80 \times 10^{-3}$$

6.4 Unit Conversions (Unit Factor Method)

Conversions are often necessary in both math and science problems. Whether you are working on a problem involving the metric system or converting moles to grams, the unit factor method is a useful tool.

The unit factor method is the sequential application of conversion factors expressed as fractions and arranged so that any dimensional unit appearing in both the numerator and denominator of any of the fractions can be cancelled out until only the desired set of dimensional units is obtained

Example 1: Convert a speed of 25 m/s to km/h.

The equivalent relationships are: 1000 m = 1 km, 60 s = 1 min, and 60 min = 1 h. Multiplying by unit factors and carefully analyzing the units results in

$$25 \frac{\cancel{m}}{\cancel{s}} \times \frac{1 \text{ km}}{1000 \cancel{m}} \times \frac{60 \cancel{s}}{1 \cancel{\text{min}}} \times \frac{60 \cancel{\text{min}}}{1 \text{ h}} = 90 \text{ km/h}$$

Example 2: 1 ng = 10^{-9} g, therefore,

$$\frac{\text{ng}}{10^{-9} \text{ g}} = 1 \text{ or } \frac{10^{-9} \text{ g}}{\text{ng}} = 1$$

This type of relation can be used to convert units from one to another.

Example 3: Convert 5.3 mL to μL.

Multiply by factors of 1 to remove the prefix "m" and to introduce "μ".

$$5.3 \cancel{\text{mL}} \times \frac{10^{-3} \cancel{L}}{\cancel{\text{mL}}} \times \frac{\mu\text{L}}{10^{-6} \cancel{L}} = 5.3 \times 10^3 \ \mu\text{L}$$

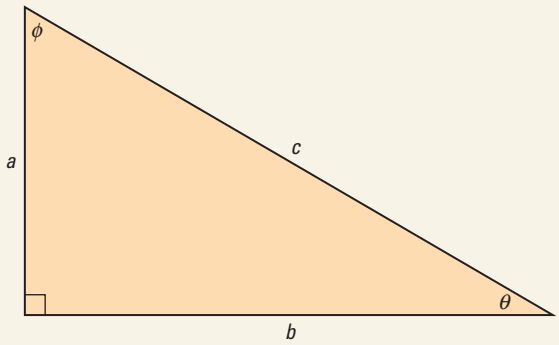
mL "cancel" as do L, leaving the desired units of μL. Subtract the exponents for division:
$-3 - (-6) = 3$

This skill is also useful for drawing scale diagrams. For example, a force of 840 N is to be drawn at a scale of 1 cm = 50 N. The scale length, in centimetres, will be found thus,

$$840 \cancel{\text{N}} \times \frac{1 \text{ cm}}{50 \cancel{\text{N}}} = 16.8 \text{ cm}$$

6.5 Trigonometry for Solving Right Triangles

A right triangle is a special triangle with one right angle (90°). The side opposite the right angle is always the longest side and is called the hypotenuse. The other two sides are called the legs. There are several important relationships that allow us to solve right triangles as long as we know the lengths of any two sides, or the length of one side and the measure of one of the acute angles.



▲ **Figure SR 6.4**

In general, the relationships are:

Angle Sum
The angles of a plane triangle add to 180°; the acute angles of a plane right triangle add to 90°.

Pythagorean Theorem
The square of the length of the hypotenuse is equal to the sum of the squares of the length of each of the legs.

$c^2 = a^2 + b^2$

Sine Ratio
Sine of one of the acute angles is equal to the ratio of the length of the leg opposite the angle to the length of the hypotenuse.

$\sin \theta = \text{opp/hyp}$
$\sin \theta = a/c$

Cosine Ratio
Cosine of one of the acute angles is equal to the ratio of the length of the leg adjacent the angle to the length of the hypotenuse.

$\cos \theta = \text{adj/hyp}$
$\cos \theta = b/c$

Tangent Ratio
Tangent of one of the acute angles is equal to the ratio of the length of the leg opposite the angle to the length of the leg adjacent to the angle.

$\tan \theta = \text{opp/adj}$
$\tan \theta = a/b$

SR 7 Tables

7.1 SI Prefixes

Prefix	Symbol	Scientific Notation
yotta-	Y	10^{24}
zetta-	Z	10^{21}
exa-	E	10^{18}
peta-	P	10^{15}
tera-	T	10^{12}
giga-	G	10^{9}
mega-	M	10^{6}
kilo-	k	10^{3}
hecto-	h	10^{2}
deka-	da	10^{1}
deci-	d	10^{-1}
centi-	c	10^{-2}
milli-	m	10^{-3}
micro-	μ	10^{-6}
nano-	n	10^{-9}
pico-	p	10^{-12}
femto-	f	10^{-15}
atto-	a	10^{-18}
zepto-	z	10^{-21}
yocto-	y	10^{-24}

7.2 Fundamental Quantities and Base Units

Quantity	Quantity Symbol	Unit	Unit Symbol
Length or Distance	ℓ	metre	m
Mass	m	kilogram	kg
Time	t	second	s
Electrical current	I	ampere	A
Thermodynamic temperature	T	kelvin	K
Quantity of matter	n	mole	mol

7.3 Derived Quantities and Units

Quantity	Quantity Symbol	Unit	Unit Symbol	Expression in terms of SI Base Units
Area	A	square metre	m^2	—
Volume	V	cubic metre	m^3	—
Speed, Velocity	v	metre per second	m/s	—
Acceleration	a	metre per second squared	m/s^2	—
Frequency	f	hertz	Hz	s^{-1}
Force	F	newton	N	$kg \cdot m/s^2$
Momentum	p	kilogram·metre per second	kg·m/s	—
Impulse	J	newton·second	N·s	kg·m/s
Energy, Work	E, W	joule	J	$kg \cdot m^2/s^2$
Power	P	watt	W	$kg \cdot m^2/s^3$
Electric charge	Q	coulomb	C	A·s
Electric potential	V	volt	V	$kg \cdot m^2/s^3 \cdot A$
Electric resistance	R	ohm	Ω	$kg \cdot m^2/s^3 \cdot A^2$
Activity	A	becquerel	Bq	s^{-1}
Absorbed dose	D	gray	Gy	m^2/s^2
Equivalent absorbed dose	E	sievert	Sv	m^2/s^2

7.4 Numerical Constants

Name	Symbol	Value
Charge on an electron	e	1.602×10^{-19} C
Gravitational constant	G	6.673×10^{-11} N·m^2/kg^2
Coulomb's constant	k	9.0×10^{9} N·m^2/C^2
Atomic mass unit	u	1.661×10^{-27} kg
Rest mass of an electron	m_e	9.109×10^{-31} kg
Rest mass of a proton	m_p	1.673×10^{-27} kg
Rest mass of a neutron	m_n	1.675×10^{-27} kg
Speed of light	c	2.9979×10^{8} m/s
Planck's constant	h	6.626×10^{-34} J·s
Rydberg's constant (hydrogen)	R_H	1.097×10^{7} m^{-1}

7.5 Atomic Masses of Selected Isotopes

Isotope	Symbol	Atomic Mass (u)
hydrogen	H	1.007 825
deuterium	H (or D)	2.014 102
tritium	H (or T)	3.016 049
helium-3	He	3.016 029
helium-4	He	4.002 603
carbon-12	C	12 (by definition)
nitrogen-16	N	16.006 102
oxygen-16	O	15.994 915
neon-20	Ne	19.992 440
neon-22	Ne	21.991 385
sodium-22	Na	21.994 436
sodium-23	Na	22.989 769
magnesium-24	Mg	23.985 042
silicon-28	Si	27.976 927
potassium-39	K	38.963 707
potassium-40	K	39.963 998
calcium-40	Ca	39.962 591
iron-56	Fe	55.934 938
iron-58	Fe	57.933 276
cobalt-60	Co	59.933 817
nickel-60	Ni	59.930 786
bromine-87	Br	86.920 711
krypton-92	Kr	91.926 156
zirconium-94	Zr	93.906 315

Isotope	Symbol	Atomic Mass (u)
tellurium-112	Te	111.917 010
tellurium-139	Te	138.934 700
cesium-140	Cs	139.905 439
barium-141	Ba	140.914 412
neodymium-144	Nd	143.910 087
lanthanum-146	La	145.925 791
lead-204	Pb	203.973 044
lead-208	Pb	207.976 652
polonium-208	Po	207.981 246
lead-210	Pb	209.984 189
polonium-212	Po	211.988 868
polonium-214	Po	213.995 201
radon-222	Rn	222.017 578
radium-226	Ra	226.025 410
thorium-230	Th	230.033 134
thorium-234	Th	234.043 601
protactinium-234	Pa	234.043 308
uranium-235	U	235.043 930
uranium-238	U	238.050 788

Note: Measurements of the atomic mass of most stable isotopes are accurate to at least a millionth of an atomic mass unit. However, the masses of highly unstable isotopes are more difficult to measure. For such isotopes, measurement errors can be large enough that the last one or two digits listed for their masses are not known for certain.